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Abstracts

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Abstracts

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Abstracts of MMA2025, May 26 - May 29, 2025, Druskininkai, Lithuania

THE READ-BAJRAKTAREVIĆ FUNCTIONAL ON A HARDY-ORLICZ SPACE

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In this talk we examine the existence of complex local fractal functions in a particular Hardy–Orlicz class. A local fractal function is the fixed point of the Read-Bajraktarević functional. The graph of the fixed point is the attractor of a determined iterated function system (IFS), whose construction is fairly standard. However, local fractal functions of the Hardy–Orlicz class enjoy of a few particular properties stemming from the complex conjugation. For example, both the fixed point and its graph are *real* in a precise sense. This is reflected as an embedding of the *realified* attractor of the induced complex iterated function system (CIFS) in the product $[1, 1] \times R$. We provide a characterization of this type of IFS via an integral version of the Read-Bajraktarević operator which appeared recently in the literature. The construction of the associated CIFS in this case requires of a local mean value theorem for holomorphic functions, which dates back to 1965.

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ON A FUZZY LOGIC BASED APPROACH TO THE VALUE OF INFORMATION IN OPTIMAL CONTROL UNDER UNCERTAINTY

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Information has an economic value because it enables decision makers to make better decisions than they could make without it. The concept of Value of Information (VoI) is based on the information theory and was initially introduced in the late 60s [1; 2] (see also [3]) and later developed in a series of works (e.g., see [4]). It constitutes a quantitative instrument to assess decisions under uncertainty related to the procurement of information. Specifically, VoI makes it possible to evaluate the profit from obtaining new information against the background of its current level.

Although VoI has become increasingly popular in recent decades (e.g., [5], [6], [7]), the classical approach has its drawbacks. Its applicability can be severely restricted in situations where stochastic modelling of the problem is difficult or infeasible (probability does not exist due to the nature of the problem). To overcome this difficulty, we propose an alternative approach based on fuzzy logic.

We develop a systematic framework for fuzzy modelling of uncertainties for decision-making, focused on further development of the theory of VoI with applications in ecological management. This includes analysis of VoI for related optimal control problems, as well as the development of decision-making procedures for assessing the need of acquiring new information.

The purpose of this talk is to introduce conference participants to the project *"A fuzzy logic based approach to the value of information estimation in optimal control problems under uncertainty with applications to ecological management,"* which is one of the research directions implemented at the Department of Mathematics of the University of Latvia in 2025–2027.

Acknowledgements. This research is funded by the Latvian Council of Science, project No. lzp-2024/1-0188.

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MULTIPLE COEXISTING OSCILLATORS

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The theory of nonlinear oscillators is an important part of the theory of ordinary differential equations. It has multiple applications in various fields of science. A single second order nonlinear differential equation can contain several coexisting oscillators in the form of period annuli. The trivial period annuli are central regions with an equilibrium of the type center. Of special interest are nontrivial period annuli that are characterized by more than one critical point inside. Oscillatory properties of equations containing multiple period annuli are studied. First, the results on the existence and location of period annuli in autonomous equations are formulated. The influence of damping terms and external periodic forces on the oscillatory properties of solutions in period annuli is in focus. The irregular behavior of solutions is recognized. Further perspectives are discussed.

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STRUCTURE-PRESERVING LEARNING OF DYNAMICAL SYSTEMS WITH DIMENSIONALITY REDUCTION

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Due to the considerable success of the development and application of structure-preserving numerical integrators [1], in recent years, great attention has also been given to the development of structure-preserving learning methods of dynamical systems [2; 3; 4], either by preserving structural properties of the dynamical system's equations or geometric properties of the dynamical system's flow. In particular, symplecticity-preserving neural networks such as SympNets have been proposed in [2] to learn the flow of a symplectic Hamiltonian dynamics, such that in the spirit of symplectic numerical methods, qualitatively better long-term predictions can be obtained.

Learning high-dimensional problems still poses a significant computational challenge. To improve computational efficiency, we have proposed learning with structure-preserving dimensionality reduction [4], such as the proper symplectic decomposition (PSD) [5], to preserve the inherent geometric property of the system when learning Hamiltonian dynamics. In addition, we have extended our results in [4] by deriving a symplecticity-preserving unconstrained parametrization of symplecticity-preserving dimensionality reduction matrices. With this unconstrained parametrization, symplectic dimensionality reduction can be learned simultaneously with learning Hamiltonian dynamics in the dimension-reduced phase space. Obtained numerical results demonstrate more accurate long-term predictions compared to learning dimension-reduced dynamics with fixed prescribed PSD solution.

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DISCRETE STURM–LIOUVILLE PROBLEM WITH TWO-POINT NONLOCAL BOUNDARY CONDITION AND NATURAL APPROXIMATION OF A DERIVATIVE

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We investigate SLP with one classical Dirichlet Boundary Condition (BC) or Neumann BC:

$$-u'' = \lambda u, \quad t \in (0,1), \quad \lambda \in \mathbb{C}, \tag{1}$$

$$u(0) = 0 \quad \text{or} \quad u'(0) = 0, \tag{$2_{d,\,n}$}$$

and another two-points Nonlocal Boundary Condition (NBC)

$$u(1) = \gamma u(\xi), \qquad u'(1) = \gamma u'(\xi), \qquad u(1) = \gamma u'(\xi) \quad \text{or} \quad u'(1) = \gamma u(\xi), \tag{$3_{1,2,3,4}$}$$

where NBC's parameter $\gamma \in \mathbb{R}$ and $\xi \in [0,1]$.

We introduce a uniform grids in $[0,1]$: $\overline{\omega}^h = \{t_j = jh, j = \overline{0,n}\}$, $\omega^h = \{t_j = jh, j = \overline{1,n-1}\}$ with stepsizes $h_j \equiv h$ and $\omega^h_{1/2} = \{t_{j+1/2} = (t_j + t_{j+1})/2, j = \overline{0,n-1}\}$ with stepsizes $h_{j+1/2} = t_{j+1/2} - t_{j-1/2} \equiv h$. Additionally, we use a nonuniform grid $\overline{\omega}^h_{1/2} = \omega^h_{1/2} \cup \{t_{-1/2} = 0, t_{n+1/2} = n\}$ where stepsizes $h_{1/2} = t_{1/2} - t_{-1/2} = h/2$, $h_{n+1/2} = t_{n+1/2} - t_{n-1/2} = h/2$. We make an assumption that $\xi = m/n$ is located on the grid $\overline{\omega}^h$. We approximate differential SLP (1)—(3) by the discrete SLP, using natural approximation of derivative $\overline{\delta}$:

$$-\delta^2 U = \lambda U, \quad t \in \omega^h, \quad \lambda \in \mathbb{C}_\lambda, \tag{4}$$

$$U_0 = 0 \quad \text{or} \quad (\overline{\delta} U)_0 = 0, \tag{$5_{d,\,n}$}$$

$$U_n = \gamma U_m, \qquad (\overline{\delta} U)_n = \gamma (\overline{\delta} U)_m, \qquad U_n = \gamma (\overline{\delta} U)_m \quad \text{or} \quad (\overline{\delta} U)_n = \gamma U_m, \tag{$6_{1,2,3,4}$}$$

We investigate discrete SLP and analyze how complex eigenvalues of this problem depend on the parameters of the two-points NBC. Some results for the both SLP were presented in [1; 3].

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FROM CHURN INSIGHTS TO SUSPICION-BASED FRAUD DETECTION IN TELECOM

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ABSTRACT

Telecom fraud detection is often framed as a binary classification problem – users are either "fraudulent" or "legitimate." Yet, for Mobile Virtual Network Operators (MVNOs), incomplete network visibility (especially limited CDR data), evolving fraud tactics, and inconsistent data availability makes a fully labeled fraud dataset practically unachievable.

In this talk, we first outline our recent research on telecom churn [1], where we employed a *sliding-window approach* to capture concept drift over time. We then draw parallels between churn and fraud prediction tasks, underscoring the additional complexities of fraud labeling:

- **Data Limitations:** MVNOs often track only outbound calls, narrowing the range of features for potential fraud indicators.
- **Label Ambiguity:** Fraudulent behavior is difficult to prove, making it hard to label as "legitimate" or "fraudulent".
- **Dynamic Patterns:** Fraud schemes (e.g., *tariff plan abuse*) evolve rapidly, further complicating a static labeling process.

Given these constraints, we propose *stepping back* from labeling outright fraud and instead focusing on *identifying suspicious behavior*. By creating a series of dataset snapshots over time – analogous to our sliding-window approach for churn – we can monitor how suspicious indicators fluctuate. This yields an adaptive, partially labeled dataset that practitioners can refine incrementally as new information emerges. While we do not present new quantitative results on fraud, we review other researchers' approaches and stress how a suspicion-oriented framework can benefit MVNOs with limited data.

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ON THE STABILITY AND EFFICIENCY OF HIGH-ORDER PARALLEL ALGORITHMS FOR 3D WAVE PROBLEMS

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In this work, we investigate the stability conditions for four new high-order ADI type schemes proposed to solve 3D wave equations with a non-constant sound speed coefficient [1; 2].

$$\frac{\partial^2 u}{\partial t^2} = c^2(X)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + f, \quad X := (x,y,z) \in \Omega, \quad t \in (0,T], \tag{1}$$

$$u(X,0) = u_0(X), \quad \partial_t u|_{t=0} = u_1(X), \quad X \in \Omega = (0,L_1)\times(0,L_2)\times(0,L_3), \tag{2}$$

$$u|_{\partial\Omega} = g(X,t). \tag{3}$$

Here $c(X)$ is the variable sound speed and it don't depend explicitly on time. We assume that

$$0 < c_m \le c(X) \le c_M, \quad \text{for} \quad X \in \Omega.$$

This analysis is mainly based on the spectral method, therefore a basic benchmark problem is formulated with a constant sound speed coefficient. For a case of general non-constant coefficient the stability analysis is done by using the energy method. Our main conclusion states that the selected ADI type schemes use different factorization operators (mainly due to the need to approximate the artificial boundary conditions on the split time levels), but the general structure of the stability factors are similar for all schemes and thus the obtained CFL conditions are also very similar.

The second goal is to compare the accuracy and efficiency of the selected ADI solvers. This analysis also includes parallel versions of these schemes. Two schemes are selected as the most effective and accurate.

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ERROR ESTIMATION OF THE DIFFERENCE SCHEME FOR ELLIPTIC EQUATION WITH AN INTEGRAL BOUNDARY CONDITION

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The boundary value problem for nonlinear elliptic equation with nonlocal integral condition depending on two parameters ξ and γ is considered:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x,y,u), \quad (x,y) \in \Omega = \{0 < x < 1,\ 0 < y < 1\},$$

$$u(x,0) = \mu_1(x), \quad u(x,1) = \mu_2(x), \quad u(0,y) = \mu_3(y), \quad u(1,y) = \gamma \int\limits_{\xi}^{1} u(x,y) + \mu_4(y),$$

where $\gamma \geq 0$, $0 \leq \xi \leq 1$, μ_1, μ_2, μ_3, μ_4 – known sufficiently smooth functions.

It was analyzed how the variation of the parameters ξ and γ changes properties of the matrix of corresponding difference equations system. Also, it was examined the influence of parameters ξ and γ for the matrix of corresponding difference equation system to be M-matrix. Properties of an M-matrix allow to estimate the error of the solution occurring from the nonlocal condition.

The main aim of investigation is to estimate the error occurring from the nonlocal condition. In [1], based on the properties of M-matrices, a statement was proved, which is commonly referred to as the comparison theorem in the theory of finite difference method. According to this theorem, a majorant function can be formed and the error is estimated using this function.

Using this function and depending on the values of parameters ξ and γ it is possible to evaluate the error of the approximate solution obtained by the finite difference method. Numerical experiment [2] was performed in order to supplement and clarify the theoretical statements on the construction of the majorant.

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ERROR ANALYSIS FOR HIGHER-ORDER METHODS FOR SUBDIFFUSION EQUATIONS ON QUASI-GRADED MESHES

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An initial-boundary value problem with a Caputo time derivative of fractional order $\alpha \in (0,1)$ is considered, solutions of which typically exhibit a singular behaviour at an initial time. I will start with a review of [1] and [2], where we give a simple and general numerical-stability analysis using barrier functions, which yields sharp pointwise-in-time error bounds on quasi-graded temporal meshes with arbitrary degree of grading. This approach was initially employed in the error analysis of the L1 method. This methodology is also generalized for semilinear fractional parabolic equations [4].

The main focus of the talk will be on higher-order discretizations, such as the Alikhanov L2-1_σ scheme, also considered in [2], and an L2-type discretization of order $3-\alpha$ in time [3]. Some more recent results for the latter will also be presented [5]. The theoretical findings are illustrated by numerical experiments.

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CURVE ESTIMATION BY A NATURAL SPLINE BASED ON REDUCED DATA

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We analyze the problem of interpolating the sequence of reduced data points $\mathcal{Q}_m = \{q_i\}_{i=0}^m = \gamma(t_i)$ in arbitrary Euclidean space $\mathbb{E}^n$. Here, contrary to the classical setting (see e.g. [1]) the corresponding knots $\mathcal{T}_m = \{t_i\}_{i=0}^m$ are not supplemented to $\mathcal{Q}_m$. In our setting, the missing interpolation knots $\mathcal{T}_m$ are compensated by their substitutes $\hat{\mathcal{T}}_m = \{\hat{t}_i\}_{i=0}^m$ (guessed from $\mathcal{Q}_m$) in accordance with the so-called *exponential parameterization* (see e.g. [2]):

$$\hat{t}_0 = 0, \qquad \hat{t}_{i+1} = \|q_{i+1} - q_i\|^\lambda, \tag{1}$$

with $i = 0, \dots, m-1$ and $\lambda \in [0,1]$. In case of dense data points $\mathcal{Q}_m$ (i.e. for $m \to \infty$) the convergence rates in approximating the curve γ by various fitting schemes $\hat{\gamma} \in C^k$ (for $k = 0,1,2$) (based on $\mathcal{Q}_m$ and (1)) were studied e.g. for $\hat{\gamma}$ representing *piecewise Lagrange quadratics* or *cubics*, *modified Hermite interpolant* or *complete spline* (see [3; 1; 2; 6]).

In this work the asymptotics in γ estimation by *a natural spline* $\hat{\gamma} = \hat{\gamma}^{NS}$ (i.e. where $\hat{\gamma}''(0) = \hat{\gamma}''(\hat{T}) = \vec{0}$) in conjunction with (1) is discussed. *A linear convergence rate* (or *a quadratic one*) is established in $\gamma \approx \hat{\gamma}^{NS}$ for $\lambda \in [0,1)$ (or $\lambda = 1$). The asymptotics derived is expressed in terms of $\delta_m = \max_{i\in\{0,\dots,m-1\}}\{t_{i+1} - t_i\}$ and coincides with $\gamma^{NS} \approx \gamma$ based on *non-reduced data* $(\mathcal{T}_m, \mathcal{Q}_m)$. Numerical tests confirm the above asymptotics and *its sharpness* for various $2D$ and $3D$ curves γ.

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ON EQUIVALENTS OF THE RIEMANN HYPOTHESIS

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Let $s=\sigma+it$ be a complex variable. The Riemann zeta-function $\zeta(s)$, for $\sigma>1$, is defined by

$$\zeta(s)=\sum_{m=1}^{\infty}\frac{1}{m^s}=\prod_p\left(1-\frac{1}{p^s}\right)^{-1},$$

where the infinite product is taken over all prime numbers.

Zeros of $\zeta(s)$ occupy an important place in mathematics. The zeros $s=-2k$, $k\in\mathbb{N}$, are called trivial. Moreover, $\zeta(s)$ has infinitely many non-trivial zeros lying in the strip $\{s\in\mathbb{C}:0<\sigma<1\}$. The Riemann hypothesis (RH) states (1859) that all non-trivial zeros of $\zeta(s)$ are on the line $\sigma=1/2$, or equivalently, that $\zeta(s)\neq 0$ for $\sigma>1/2$. The RH is the eight Hilbert problem (1900), and is among the most important seven Millennium problems of mathematics.

There are many various equivalents of RH. In the report, we propose some equivalents in terms of self approximation.

Let $\Gamma(s)$ denote the Euler gamma-function, and $\theta(t)$ be the increment of the argument of the function $\pi^{s/2}\Gamma(s/2)$ along the segment connecting the points $s=1/2$ and $s=1/2+it$. Then the equation

$$\theta(t)=\pi(k-1),\quad k\in\mathbb{N},$$

has the unique solution t_k called the Gram point.

Let $D=\{s\in\mathbb{C}:1/2<\sigma<1\}$, and $\mathcal{K}$ be the class of compact subsets of D with connected complements.

THEOREM 1. *The RH is equivalent to each of the assertions:*

1° *For every $K\in\mathcal{K}$ and $\varepsilon>0$,*

$$\liminf_{N\to\infty}\frac{1}{N}\#\left\{1\leqslant k\leqslant N:\sup_{s\in K}|\zeta(s+it_k)-\zeta(s)|<\varepsilon\right\}>0.$$

2° *For every $K\in\mathcal{K}$, the limit*

$$\lim_{N\to\infty}\frac{1}{N}\#\left\{1\leqslant k\leqslant N:\sup_{s\in K}|\zeta(s+it_k)-\zeta(s)|<\varepsilon\right\}$$

exists and is positive for all but at most countably many $\varepsilon>0$.

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MATHEMATICAL ANALYSIS OF WASTE REDUCTION INDICATORS

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The generation and management of municipal waste and the recycling of waste and bio-waste are relevant parameters in EU countries. These parameters contribute to the overall well-being and sustainability of the environment. In order to understand and analyze the situation of waste management in 27 EU countries we used well known clustering methods. We analyzed data related to municipal waste, waste recycling and biowaste recycling in the EU countries, collected between 2000 and 2021. Calculations were made using the R Statistical Software [1]. All countries were grouped in several clusters [2] using Partitioning Around Medoids (PAM) algorithm. The agglomerative clustering methods were used for the better understanding of clusters formation and the countries forming clusters were compared using the nonparametric Kruskal-Wallis test. During the research process we also payed attention to the influence of outliers. The analysis showed that five EU countries (Austria, Denmark, Germany, Luxembourg, and the Netherlands) are always grouped into the same cluster based on the three waste variables examined. Such compliance requires uniform waste management practices or similar waste properties in these countries. Since all three variables are correlated with each other and the correlations are not weak, we used factor analysis [3], which allows us to identify hidden variables and to determine the main components. After principal component analysis (PCA) it was concluded that two factors describe the countries quite well. They describe the municipal waste very well (explain more than 90% of the variation) and part of the variation in both recycling of waste and bio-waste. To better identify factors, we use factor rotation. After rotation both factors explain 95 percent of common variable variations. The performed data analysis implies a general trend that the studied countries are actually separated from each other, either by a single "waste" factor that includes municipal waste or a "recycling" factor that includes recycling of waste and bio-waste.

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A NEW JOINT LIMIT THEOREM FOR EPSTEIN ZETA-FUNCTIONS

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In [1], we began to characterize the asymptotic behaviour of the Epstein zeta-function $\zeta(s;Q)$ using the Bohr-Jessen method [2] and techniques developed by Antanas Laurinčikas [3].

Let Q be a positive definite quadratic $n\times n$ matrix and $Q[\underline{x}] = \underline{x}^{\mathrm{T}}Q\underline{x}$ for $\underline{x}\in\mathbb{Z}^n$. The Epstein zeta-function $\zeta(s;Q)$, where $s=\sigma+it$, is defined by the series $\zeta(s;Q)=\sum_{\underline{x}\in\mathbb{Z}^n\setminus\{\underline{0}\}}(Q[\underline{x}])^{-s}$, for $\sigma>\frac{n}{2}$, and can be continued analytically to the whole complex plane, except for a simple pole at $s=\frac{n}{2}$, with residue $\frac{\pi^{n/2}}{\Gamma(n/2)\sqrt{\det Q}}$. In fact, the function $\zeta(s;Q)$ represents a class of Dirichlet series depending on the matrix Q. This class is quite general and provides meaningful results. In [1], we have proved that, on $(\mathbb{C},\mathcal{B}(\mathbb{C}))$, there exists an explicitly given probability measure $P_{Q,\sigma}$ such that, for even $n\geq 4$, $Q[\underline{x}]\in\mathbb{Z}$, and $\sigma>\frac{n-1}{2}$, the limit measure $\frac{1}{T}\mathrm{meas}\big\{t\in[0,T]:\zeta(\sigma+it;Q)\in A\big\}, A\in\mathcal{B}(\mathbb{C})$, converges weakly to $P_{Q,\sigma}$ as $T\to\infty$. Here $\mathrm{meas}\{A\}$ denotes the Lebesgue measure of a measurable set $A\subset\mathbb{R}$, and $\mathcal{B}(\mathbb{C})$ is the Borel σ-field of the space $\mathbb{C}$.

A joint limit theorem for a collection of the Epstein zeta-functions has been obtained in [4]. For $j=1,\dots,r$, let Q_j denote a positive definite quadratic $n_j\times n_j$ matrix, and $\zeta(s_j;Q_j)$ be the corresponding Epstein zeta-function. Denote by $\underline{s}=(s_1,\dots,s_r)$, $\underline{Q}=(Q_1,\dots,Q_r)$ and $\underline{\zeta}(\underline{s};\underline{Q})=(\zeta(s_1;Q_1),\dots,\zeta(s_r;Q_r))$. Thus, in [4], we showed that for fixed $\sigma_j>\frac{n_j-1}{2}$, $j=1,\dots,r$, $\frac{1}{T}\mathrm{meas}\left\{t\in[0,T]:\underline{\zeta}(\underline{\sigma}+it;\underline{Q})\in A\right\}$, $A\in\mathcal{B}(\mathbb{C}^r)$, converges weakly to an explicitly given probability measure as $T\to\infty$.

This talk focuses on a generalization of the mentioned result. We will discuss the weak convergence of

$$\frac{1}{T}\mathrm{meas}\left\{t\in[T,2T]:\underline{\zeta}(\underline{\sigma}+i\underline{\varphi}(t);\underline{Q})\in A\right\},\quad A\in\mathcal{B}(\mathbb{C}^r),$$

for $\underline{\varphi}(t)=\big(\varphi_1(t),\dots,\varphi_r(t)\big)$, where the functions $\varphi_1(t),\dots,\varphi_r(t)$ are increasing to $+\infty$, have monotonic derivatives $\varphi_j'(t)$ satisfying $\varphi_j(t)\ll t\varphi_j'(t)$, and $\varphi_j'(t)=o(\varphi_{j+1}'(t))$ as $t\to\infty$.

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ON MULTI-SCALE MODELLING AND DECOMPOSITION TECHNIQUES FOR AUTO-IGNITION IN BIOMASS FLOW SYSTEMS

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The prevention of auto-ignition in biomass storage and processing environments is a critical challenge due to the risk of thermal runaway initiated by particle deposition and chemical reactivity. We propose a modelling framework that integrates fluid flow, particle transport, heat generation, and combustion kinetics.

In our approach, biomass particles are tracked individually in a Lagrangian fashion within a precomputed velocity field [1]. Upon collision with obstacles or other particles, they become immobilized, forming localized deposition sites. Each deposition triggers a local heat pulse, and repeated collisions within short time intervals can lead to thermal build-up.

The evolution of temperature and species concentrations (e.g., CH_4, O_2, CO_2) is governed by a coupled system:

$$\rho c_p \frac{DT}{Dt} = \nabla \cdot (k \nabla T) + \sum_i (-\Delta H_i \dot{\omega}_i), \qquad \frac{DC_i}{Dt} = \nabla \cdot (D_i \nabla C_i) + \dot{\omega}_i$$

with Arrhenius-type reaction rates [2].

To reduce computational cost, we implement a modular decomposition: (i) Eulerian fluid flow computation, (ii) Lagrangian particle tracking and deposition registration, (iii) localized ignition modelling by ODEs, and (iv) an artificial diffusion module to emulate spatial heat propagation and interaction between deposition zones.

Since there's an uncertainty in obstacle layouts and particle initialization, we also discuss some model reduction strategies, such as PGD or PGD–NN hybrid [3], providing mechanisms for faster auto-ignition prediction.

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MODELLING MHD FLOW IN A THICK-WALLED PIPE SUBJECT TO ORIENTED MAGNETIC FIELDS

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The present paper considers the flow of an electrically conducting fluid through a thick-walled pipe under arbitrarily oriented applied magnetic fields. The nondimensional steady magnetohyrodynamic (MHD) flow equations, [1] are of convection-diffusion type and form a coupled system of velocity (V) and magnetic field (B) variables inside the pipe. Besides, since the pipe wall thickness is not neglected in this work, a new equation arises to model the magnetic field effect on the solid boundary of the pipe. This causes additional unknowns and a two layered fluid-wall problem domain with a fluid to solid transition interface. Thus, uncoupling the resulting system becomes more difficult and even if it is succeeded still the boundary conditions remain coupled.

In this study, a semi analytic boundary element method (BEM) is derived newly for the solution of the system through (1)-(2) given in [2].

$$\left.\begin{aligned} &\nabla^2 V + ReR_h \left(\frac{\partial B_1}{\partial x} \sin\alpha + \frac{\partial B_1}{\partial y} \cos\alpha \right) = -1 \\ &\nabla^2 B_1 + R_{m_1} \left(\frac{\partial V}{\partial x} \sin\alpha + \frac{\partial V}{\partial y} \cos\alpha \right) = 0 \end{aligned}\right\} \text{in } \Omega_1 \quad \text{and} \quad \nabla^2 B_2 = 0 \quad \text{in } \Omega_2 \tag{1}$$

together with the hydrodynamic and the electromagnetic boundary conditions below

$$V = 0, \quad B_1 = B_2, \quad R_{m_2} \frac{\partial B_1}{\partial n} = R_{m_1} \frac{\partial B_2}{\partial n} \quad \text{on } \Gamma_1 \quad \text{and} \quad B_2 = 0 \quad \text{on } \Gamma_2 \tag{2}$$

where Ω_1 and Ω_2 are the fluid and wall regions, respectively with the inner and outer boundaries Γ_1 and Γ_2. The angle α denotes the orientation of the external magnetic field. With the new approach, the equations are treated as a whole without the need of a decoupling procedure. The existing fundamental solution for the coupled convection-diffusion type MHD equations in [3] is utilized and so the convection-dominance is preserved. Moreover, the coupled boundary conditions are imbedded analytically in a novel way and a square system is finally derived. A detailed numerical assessment is performed to observe the effect of the orientation angle and pipe wall thickness. The computational results show the hydromagnetic characteristics of the flow very well.

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QUANTIFYING MEMORY-INDUCED CHAOS: COMPLEXITY, WADA INDEX, AND ARNOLD TONGUES

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Fractional calculus, with its inherent memory-preserving properties, has offered profound insights into the behavior of nonlinear discrete systems. This study explores how the introduction of fractional derivatives—specifically in the Caputo sense—reshapes the complexity and dynamical structure of the standard map, a foundational system in chaos theory.

The first part of this investigation is centered on the scalar Caputo fractional standard map, where the fractionality parameter $\alpha \in (1, 2]$ directly influences the system's transition from order to chaos. As α approaches 1, the system exhibits intricate Arnold tongue patterns, reflecting a rich tapestry of bifurcations and resonances [1].

To analyze this transition, we adopt the Wada index, a measure originally developed to classify the interwoven structure of basins of attraction [2]. In addition, a novel complexity parameter is introduced to quantify the degree of unpredictability of the system. Both measures provide complementary perspectives: while the Wada index captures topological intricacy, the complexity parameter reflects the dynamical variability of the trajectories. These tools reveal a nuanced dependence on the fractionality parameter, where lower values of α lead to increased complexity and intricate basin structures.

Building on these insights, the work is extended into a higher-dimensional context by replacing scalar iterative variables with matrix-valued variables, thus defining the Caputo fractional standard map of matrices [3]. This extension opens a new avenue in the analysis of multidimensional fractional systems, specifically focusing on nilpotent matrices, which induce finite-time or even explosive divergence. The divergence behavior is not just a continuation of scalar dynamics but rather a fundamental shift that results in the formation of Arnold tongues of divergence.

The present study bridges the scalar and matrix realms of fractional maps, illuminating how memory and algebraic structure intertwine to give rise to rich dynamical behavior in both theoretical and applied contexts.

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APPROXIMATION OF EIGENVALUES OF THE DIFFUSION OPERATOR IN DOMAINS CONTAINING THIN TUBES BY ASYMPTOTIC PARTIAL DOMAIN DECOMPOSITION

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We consider the problem on the spectrum of diffusion operator in a domain containing thin tubes with Neumann boundary condition on the lateral boundary of tubes. The problem is approximated by a spectral problem of hybrid dimension, where the tubes are replaced by one-dimensional segments with one-dimensional diffusion operator on these segments. The junction conditions on the interfaces of 1D and 3D parts of the domain are : pointwise continuity of the flux and continuity in average of the eigenfunctions, so that it can be discontinuous. The error estimates are proved for the difference of the eigenvalues of the original and approximate problems. The presented results are obtained in collaboration with A.Amosov, D. Gomez, and M.E. Perez-Martinez.

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A FUZZY TRANSFORM APPROACH TO WEAK SOLUTIONS IN SINGULAR VOLTERRA EQUATIONS

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F^0-transform and F^1-transform [2; 3] are powerful methods of fuzzy modeling for approximate functions from L^2. There applications are very wide in many fields such as: time series, neural network, etc. Based on the preceding analysis [4], we employ the F^0-transform and F^1-transform to compute approximate solutions for the weakly singular Volterra integral equation (1).

The general form of the Volterra integral equation of the second kind is:

$$y(t) = \int_0^t K(t,s)y(s)ds + g(t), \quad t \in [0,1], \tag{1}$$

where $K : [0,1] \times [0,1] \to \mathbf{R}$ is a given kernel, g, y are given *source* and unknown functions, respectively, both mapping $[0,1]$ to $\mathbf{R}$. The equation (1) is "*weakly singular*" if the only propery of its kernel K is *absolute integrability* with respect to s, i.e.

$$\sup_{t\in[0,1]} \int_0^1 |K(t,s)|ds < \infty. \tag{2}$$

Following [1], we consider the class $S^{m,\nu}$ of kernels defined as functions that are m-times continuously differentiable ($m \geq 0$) on $[0,1] \times [0,1]$ without the diagonal, and satisfy

$$\left| \left(\frac{\partial}{\partial t}\right)^k K(t,s) \right| \leq c_{K,m}|t-s|^{-k-\nu},\ 0 \leq k \leq m,$$

where $0 < \nu < 1$. The kernels from $S^{m,\nu}$ satisfy (2) and hence they are weakly singular.

In our contribution, we will use the weakly singular kernels from $S^{1,\nu}$. It is known [1] that for all kernels from $S^{1,\nu}$ and all source functions $g \in C[0,1]$, the equation 1 has a unique solution in $C[0,1]$, and if $g \in C^{1,\nu}(0,1]$ and a solution $u \in C[0,1]$ is unique, then $u \in C^{1,\nu}(0,1]$ (Here $C^{1,\nu}(0,1]$ is a *weighted space* of function from $C^1(0,1]$, [1]). Since analytical solutions are rarely available for such problems, developing numerical methods becomes essential. Our approach applies the F^0-transform (F^1-transform) to the weakly singular Volterra operator in (1), yielding an operational matrix representation. As a result, the entire equation can be reduced to the system of linear agebraic equations with an invertible matrix. The resulting numerical scheme is both computationally efficient and inexpensive to implement. We provide rigorous theoretical support for this method, including: proof of convergence, computational complexity estimates, a discussion of connections to the Galerkin method.

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ON STABILITY OF PERIODIC MOTION OF THE SWINGING ATWOOD MACHINE

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The swinging Atwood machine under consideration is a conservative mechanical system with two degrees of freedom and its equations of motion may be written in the form

$$r\ddot{\varphi} = -\sin\varphi - 2\dot{r}\dot{\varphi}, \qquad (2+\varepsilon)\ddot{r} = -\varepsilon - (1-\cos\varphi) + r\dot{\varphi}^2. \tag{1}$$

Here the variables r, φ describe geometrical configuration of the system, and parameter $\varepsilon = (m_2 - m_1)/m_1$ determines a difference of two masses $m_1 \leq m_2$ attached to opposite ends of a massless inextensible thread (see [1]).

Note that differential equations (1) are essentially nonlinear and their solution cannot be written in symbolic form, in general. Numerical analysis of the system shows that it may demonstrate different kinds of motion. In particular, one can choose such initial conditions that motion of the system is periodic (see [2]). The corresponding solution to (1) can be represented in the form of power series in terms of small parameter ε

$$\begin{aligned}
\varphi(t) &= \sqrt{\varepsilon}\left(2\sin(\omega t) + \frac{53\varepsilon}{192}\sin(3\omega t) + \frac{\varepsilon^2}{81920}\big(14795\sin(\omega t) - 8495\sin(3\omega t) + 5813\sin(5\omega t)\big)\right),\\
r(t) &= 1 + \frac{3\varepsilon}{8}(1-\cos(2\omega t)) + \frac{\varepsilon^2}{2048}\big(69 + 36\cos(2\omega t) - 105\cos(4\omega t)\big),\\
\omega &= 1 - \frac{5\varepsilon}{32} + \frac{35\varepsilon^2}{1024} - \frac{867\varepsilon^3}{131072}.
\end{aligned} \tag{2}$$

In the case of equal masses ($\varepsilon = 0$) solution (2) degenerates into equilibrium position of the system $\varphi = 0, r = 1$ that is unstable. As for $\varepsilon > 0$ the system has no equilibrium state, the periodic motion described by solution (2) may be considered as a state of its dynamic equilibrium. An interesting peculiarity of this state is that owing to oscillations the smaller mass m_1 can counterweight the larger mass m_2 what is not possible in the absence of oscillations. In the present talk, we show that small perturbations of the initial conditions determining the periodic solution (2) lead only to small oscillations of the system near equilibrium. It gives an example of mechanical system, where the state of dynamical equilibrium is stabilized by means of oscillations.

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ON STABILITY AND CONVERGENCE OF A DIFFERENCE SCHEME FOR 2D NONLINEAR PARABOLIC EQUATION WITH NONLOCAL BOUNDARY CONDITIONS

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We construct finite difference method for nonlinear two-dimensional parabolic equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(u) + p(x, y, t) \tag{1}$$

in the rectangular domain $(x, y) \in \Omega = \{0 < x < 1, 0 < y < 1\}$ and $t \in (0, T]$ with local and nonlocal boundary conditions

$$u(1, y, t) = \gamma u(0, y, t), \tag{2}$$

$$\frac{\partial u(1, y, t)}{\partial x} = 0, \tag{3}$$

with Dirichlet type nonlocal boundary conditions

$$u(x, 0, t) = \mu_1(x, t), \tag{4}$$

$$u(x, 1, t) = \mu_2(x, t) \tag{5}$$

and with initial condition

$$u(x, y, 0) = \phi(x, y). \tag{6}$$

We construct and anlyze the bacward Euler difference scheme. For the analysis of the spectrum structure of the difference operator an M-matrix theory is used. Using this approach, the stability and convergence of the difference scheme in the maximum norm are proven. we present some new conclusions in the new approach.

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MODELING AND ANALYSIS OF NONLINEAR DYNAMICS IN SEMICONDUCTOR LASERS

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Many dynamical semiconductor laser models can be formally written as

$$\frac{d}{dt}\Psi = \mathcal{H}(\beta)\,\Psi + F_{sp}, \quad \frac{d}{dt}N = \mathcal{N}(N, |\Psi|^2), \qquad \beta = \beta_0 + \delta_\beta(N, |\Psi|^2),$$

where Ψ and N represent slowly varying complex optical field amplitudes and carrier density, respectively. The function $\mathcal{N}$ describes the evolution of carriers, while $\mathcal{H}$ is an optical field operator, which may include time delays (as in Lang-Kobayashi-type models) or spatial derivatives (as in Traveling Wave models). At each time instant, the propagation factor β and operator $\mathcal{H}(\beta)$ define a set of optical modes $[\Theta, \Omega]$, where Θ and Ω are the eigenfunctions and complex eigenfrequencies, respectively [1]. These modes can be utilized to find steady states where $\Im\Omega(\beta)$ vanishes [2], decompose the calculated optical field into modal components [1; 3], or construct reduced *mode approximation* systems of ODEs [4], relying on the evolution of only a few mode amplitudes:

$$i\Omega\Theta = \mathcal{H}(\beta)\,\Theta \quad \Rightarrow \quad \Psi(t) = \sum\nolimits_j f_j(t)\Theta_j(\beta) \quad \Rightarrow \quad \frac{d}{dt}f_k = i\Omega_k(\beta)f_k - \sum\nolimits_l K_{k,l}(\beta)f_l + \zeta_{\mathrm{sp}}^{(k)}.$$

Calculating the modes [1; 5], finding steady states [2] and tracing them as parameters are tuned [6], analyzing time-frequency representations accessible via mode expansion of the fields [3], and examining the reduced mode approximation systems [4], along with direct simulations of the original model equations, have proven to be powerful methods. These approaches provide deep insights into the dynamics of laser devices and aid in designing novel laser systems. In this talk, the advantages of mode analysis will be illustrated with examples from recent studies on external feedback [3] and photonic crystal surface-emitting semiconductor lasers [5].

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FINITE-TIME STABILIZATION OF THE FIXED POINTS OF THE FRACTIONAL DIFFERENCE MAPS

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The governing iterative model of the fractional difference maps reads [1]:

$$x_k = x_0 + \frac{1}{\Gamma(\nu)} \sum_{j=1}^{k} \frac{\Gamma(k-j+\nu)}{\Gamma(k-j+1)} (f(x_{k-j}) - x_{k-j}) \tag{1}$$

where $k = 1, 2, ...$; Γ is the Gamma function; ν is the order of the fractional derivative usually assumed $0 < \nu \leq 1$; f is the kernel corresponding to the mapping function of the non-fractional counterpart system.

Although it is well known that periodic orbits of fractional difference maps do not exist [2], this talk delves into two important aspects of period-1 orbits of fractional difference maps.

The first aspect is related to the properties of the unstable fixed points in the fractional difference maps [3]. Analytical and computational techniques are used to deduce whether a finite-time stabilization of the unstable fixed point is due to inherent physical properties of the system or a computational artifact caused by the finite precision.

The second aspect is related to the continuous adaptive stabilization of unstable orbits of fractional difference maps [4]. An impulse-based control technique without short oscillatory transients right after the control impulse is designed for the fractional difference maps with a long memory horizon. It is demonstrated that the coordinate of the unstable period-1 orbit may drift due to the continuous application of the impulse-based control scheme. An adaptive scheme capable of tracking the drifting coordinate of the unstable period-1 orbit is designed and validated by a number of computational experiments.

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RECENT PROGRESS IN THE STUDY OF WILSON–COWAN SYSTEM

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The Wilson–Cowan system was primarily invented to study two interacting populations of neurons. Its simplified version

$$\begin{cases} x_1' = \dfrac{1}{1+e^{-\mu_1\,(w_{11}x_1+w_{12}x_2-\theta_1)}} - v_1x_1, \\ x_2' = \dfrac{1}{1+e^{-\mu_2\,(w_{21}x_1+w_{22}x_2-\theta_2)}} - v_2x_2 \end{cases} \tag{1}$$

is known to have rich dynamics. The higher dimensional versions of the system (1) were adapted to model genetic networks, and similar networks in other fields. The three-dimensional version of system (1) contains 18 parameter and the number of parameters increases along with the dimensionality. The central point in the study of this system is to gather information about attractors in the phase space. The dynamics of solutions and evolution of the system heavily depend on the number, locations, and properties of attractors. In the proposed talk recent contributions to the theory are reported concerning types, properties and forms of attractors in three-dimensional version of the system (1). In particular, a collection of attractors of different shapes is presented.

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ON APPROXIMATION BY PERIODIC ZETA-FUNCTIONS IN SHORT INTERVALS

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Let $s = \sigma + it$ be a complex variable, and $\mathfrak{a} = \{a_m; m \in \mathbb{N}\}$ be a periodic sequence of complex numbers with minimal period $q \in \mathbb{N}$. The periodic zeta-function $\zeta(s; \mathfrak{a})$, for $\sigma > 1$, is defined by the Dirichlet series

$$\zeta(s; \mathfrak{a}) = \sum_{m=1}^{\infty} \frac{a_m}{m^s}.$$

Moreover, $\zeta(s; \mathfrak{a})$ has analytic continuation to the whole complex plane, except for the point $s = 1$ which is a simple pole with residue $r \stackrel{\text{def}}{=} \frac{1}{q} \sum_{m=1}^{q} a_m$. If $r = 0$, then $\zeta(s; \mathfrak{a})$ is an entire function.

Let $D = \{s \in \mathbb{C} : 1/2 < \sigma < 1\}$. It is known that, under certain conditions on sequence $\mathfrak{a}$, the shifts $\zeta(s + i\tau; \mathfrak{a})$, $\tau \in \mathbb{R}$, approximate wide classes of analytic functions defined in the strip D, see [1; 2; 3]. It is proved that the set of approximating shifts has a positive lower density in the intervals of length T as $T \to \infty$.

In the report, we propose a certain improvement of approximation by shifts $\zeta(s + i\tau; \mathfrak{a})$, i. e., we consider approximation in interval $[T, T + H]$ with $H = o(T)$ as $T \to \infty$.

Denote by $H(D)$ the space of analytic on D functions endowed with the topology of uniform convergence on compacta, and by meas$A$ the Lebesgue measure of a set $A \subset \mathbb{R}$. Then the following statement is valid.

THEOREM 2. *Suppose that $T^{27/82} \leqslant H \leqslant T^{1/2}$. Then there exists a non-empty closed set $F_{\mathfrak{a}} \subset H(D)$ such that, for every compact set $K \subset D$, function $f(s) \in F_{\mathfrak{a}}$ and $\varepsilon > 0$,*

$$\liminf_{T \to \infty} \frac{1}{H} \text{meas} \left\{ \tau \in [T, T + H] : \sup_{s \in K} |\zeta(s + i\tau; \mathfrak{a}) - f(s)| < \varepsilon \right\} > 0.$$

Moreover, "$\liminf$" can be replaced by "$\lim$" for all but at most countably many $\varepsilon > 0$.

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ADVANCES IN PHYSICS-INFORMED NEURAL NETWORKS

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In recent years, deep neural networks have demonstrated remarkable success in such diverse fields as computer vision, natural language processing, game theory, revolutionizing approaches to categorization, pattern recognition, and regression tasks. Among recent developments in scientific machine learning, Physics-Informed Neural Networks (PINNs) have emerged as a powerful framework for solving ordinary and partial differential equations using sparse data.

Since their introduction in 2017 [1], significant progress has been made in optimizing PINNs through advancements in network architectures, adaptive refinement, domain decomposition, and adaptive activation functions. A notable innovation is the Physics-Informed Kolmogorov-Arnold Networks (PIKANs) [2], which build on Kolmogorov's representation theory to offer an alternative to conventional PINNs.

In this presentation, we discuss recent PINN advancements, focusing on architectural improvements, feature expansion, optimization strategies, uncertainty quantification, and theoretical foundations. We also examine existing computational frameworks and software tools [3] solving several problems from fluid mechanics and chemical engineering.

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M-MATRICES AND DISCRETE PROBLEMS WITH NONLOCAL BOUNDARY CONDITIONS

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We uses the finite difference method for elliptic problems with nonlocal boundary conditions. We study the case where the matrix of the resulting system of linear equations is an M-matrix. We present the main properties of monotone matrices and of M-matrices. The finite difference method is considered for the two-dimensional Poisson equation with Dirichlet boundary condition, for the same one-dimensional problem with two integral conditions. Sufficient conditions for the problem to be described by an M-matrix or a monotone matrix is formulated using parameters of nonlocal boundary conditions. Another types of nonlocal boundary conditions are investigated in one-dimensional case.

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FINITE-DIFFERENCE SCHEME FOR TWO-DIMENSIONAL POISSON EQUATION WITH THE MULTIPLE INTEGRAL BOUNDARY CONDITION

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We consider the Poisson equation with nonlocal boundary condition involving multiple integral:

$$L(u) := -\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = f(x,y), \quad (x,y) \in \Omega \subset \mathbb{R}^2, \tag{1}$$

$$u(x,y) = \int_\Omega k(x,y,\xi,\eta)u(\xi,\eta)d\xi d\eta + g(x,y), \quad (x,y) \in \partial\Omega, \tag{2}$$

where $\Omega := \Omega_x \times \Omega_y$, $\Omega_x := \{x\colon 0 < x < a\}$, $\Omega_y := \{y\colon 0 < y < b\}$ [2]. We will assume that the function k satisfies the condition $\int_\Omega |k(x,y,\xi,\eta)| d\xi d\eta \le \varrho < 1$. The finite difference method for the linear two-dimensional parabolic equation with the boundary condition (2) in the square domain $\Omega = [0,1]^2$ was studied in [1].

Let us write the finite-difference scheme (FDS) for problem (1)–(2):

$$\begin{aligned} L^h(U) : &= -\delta_x^2 U - \delta_y^2 U = F, \quad (x_i,y_j) \in \omega^h, \\ U_{ij} &= [K_{ij}, U]^{\mathrm{tr}} + G_{ij}, \quad (x_i,y_j) \in \partial\overline{\omega}^h, \end{aligned}$$

and we use two-dimensional trapezoidal rule $[K_{ij}, U]^{\mathrm{tr}}$, where $K_{ij}^{lm} = k(x_i,y_j,x_l,y_m)$, $(x_i,y_j) \in \partial\overline{\omega}^h$, $(x_l,y_m) \in \overline{\omega}^h$ for approximation of integral in boundary condition. We suppose that

$$\max_{x_{ij} \in \partial\overline{\omega}^h} [|K_{ij}|, 1]^{\mathrm{tr}} \le \rho < 1.$$

The main aims of our study are the investigation of FDS for various cases of kernel K. The main difficulty of this problem is that in the non-classical case we cannot use the method of separation of variables and decompose the problem into one-dimensional problems.

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VIRTUAL MACHINE PLACEMENT BALANCING IN CLOUD COMPUTING WITH RESOURCE OVERBOOKING

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Virtual Machine (VM) placement balancing refers to the process of efficiently distributing VMs across the available physical hosts (servers) in a virtualized infrastructure. The goal is to ensure optimal resource utilization, reduce potential bottlenecks, and maintain system stability while adhering to constraints such as CPU, memory, storage, network performance, and power consumption.

Overbooking is a resource management strategy used in cloud computing where a cloud provider allocates more virtual resources (such as CPU, memory, or network bandwidth) to virtual machines (VMs) than the actual physical capacity available on the host.

VM placement balancing involves solving complex optimization problems, often modeled as bin-packing scenarios, which are NP-hard. This complexity makes it difficult to develop algorithms that can find optimal solutions in real time, especially as the scale of data centers increases [1]. Moreover, cloud environments experience fluctuating workloads, with VMs frequently changing resource demands or migrating between physical machines. This dynamism requires continuous monitoring and real-time adjustments to maintain optimal resource utilization and performance [2]. The presence of the overbooking factor significantly complicates the load balancing process. In this presentation, we will examine the modeling of the overbooking effect.

The most common method for addressing the VM placement balancing problem is heuristic-based. These solutions usually rely on a limited set of input parameters, chosen to suit the specific objective function. In this presentation, we discuss the importance of selecting appropriate input parameters for a properly defined objective function. Using synthetic data, we highlight several limitations of some VM placement balancing methods and suggest possible improvements. All experiments were conducted using the CloudSim framework, which ensures repeatable and predictable results [3].

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OPERATOR-BASED APPROACH FOR SOLVING CAPUTO FRACTIONAL DIFFERENTIAL EQUATIONS

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Fractional differential equations (FDEs) are increasingly vital for modeling complex systems with memory effects across diverse scientific fields [1]. This study presents a novel operator-based methodology designed to construct solutions to Caputo-type FDEs involving the operator $({}^{C}D^{(1/n)})^k$. This approach generalizes previous techniques presented by the authors that were limited to simpler fractional operator structures, such as the $k = 1$ case [2].

The core of the proposed framework relies on fractional power series representations of solutions. It is demonstrated that an initial FDE of the form

$$\left({}^{C}D^{(1/n)}\right)^{k} y = F(x, y) \tag{1}$$

can be transformed into a related, higher-order FDE:

$$\left({}^{C}D^{(1/n)}\right)^{kn} y = G(x, y) + u_y^{(n)}(x), \tag{2}$$

where the function G and the additional fractional power series term $u_y^{(n)}(x)$ are derived from $F(x, y)$ and the initial conditions. A key result is that the obtained FDE of type $({}^{C}D^{(1/n)})^{kn}$ can subsequently be reduced to a k-th order ordinary differential equation (ODE) by employing the nonlinear variable substitution $t = \sqrt[n]{x}$:

$$\frac{\mathrm{d}^k \widehat{y}}{\mathrm{d}t^k} = H\left(t, \widehat{y}, \frac{\mathrm{d}\widehat{y}}{\mathrm{d}t}, ..., \frac{\mathrm{d}^{k-1}\widehat{y}}{\mathrm{d}t^{k-1}}\right), \quad \text{where } y(x) = \widehat{y}(\sqrt[n]{x}). \tag{3}$$

This transformation recasts the fractional problem into the domain of ODEs, allowing the application of well-established analytical or numerical techniques for ODEs. The effectiveness and validity of this operator-based approach are illustrated through application to a fractional Riccati-type differential equation.

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NUMERICAL APPROXIMATION FOR SYSTEMS OF FRACTIONAL DIFFERENTIAL EQUATIONS – A CENTRAL PART INTERPOLATION APPROACH

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We consider the following system of linear fractional differential equations:

$$(\mathrm{D}^{\alpha}_{Cap} y_i)(t) + \sum_{j=1}^{n} a_{ij}(t) y_j(t) = f_i(t), \quad 0 \le t \le 1, \quad i = 1, \dots, n, \quad n \in \mathbb{N} := \{1, 2, \dots\}, \tag{1}$$

$$y_i(0) = y_{0i}, \quad y_{0i} \in \mathbb{R} := (-\infty, \infty), \quad i = 1, \dots, n, \tag{2}$$

where $0 < \alpha < 1$ and $\mathrm{D}^{\alpha}_{Cap} y$ is the α-order Caputo fractional derivative of a function $y = y(t)$. By assuming that the functions a_{ij}, f_i are continuous, that is, $a_{ij}, f_j \in C[0,1]$ for $i, j = 1, \dots, n$, we can show that the problem (1)-(2) possesses a unique solution $(y_1, \dots, y_n)$ such that $y_i \in C[0,1]$, $\mathrm{D}^{\alpha}_{Cap} y_i \in C[0,1]$, $i = 1, \dots, n$. Counterintuitively, however, higher smoothness assumptions on a_{ij}, f_i are not sufficient to guarantee the smoothness of the exact solution. More precisely, it can be shown that, for $a_{ij}, f_i \in C^q[0,1]$ ($q \in \mathbb{N}$), in general no more than $y_i \in C^{q,1-\alpha}(0,1]$ can hold. Here, by $C^{q,\mu}(0,1]$ ($q \in \mathbb{N}$, $0 < \mu < 1$) we denote the set of functions $y \in C[0,1] \cap C^q(0,1]$ such that

$$|y^{(k)}(t)| \le c\, t^{1-\mu-k}, \quad 0 < t \le 1, \quad k = 1, \dots, q,$$

where c is a positive constant independent of t. Singular behaviour of the exact solution near the origin is therefore expected for fractional differential equations, and necessitates careful consideration when constructing numerical methods for their solution.

In this contribution we propose a high-order method for solving (1)-(2) based on improving the boundary behavior of the exact solution with the help of a smoothing transformation, and on central part interpolation by polynomial splines on the uniform grid. The central part interpolation approach has previously been used for solving scalar linear fractional differential equations [2] and has shown accuracy and numerical stability advantages compared to more typical spline collocation methods, including collocation at Chebyshev points [3]. We apply this approach for solving (1)-(2) and derive global error estimates for the approximate solution analogously to [2].

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